Analysis of the Method for the Flat-Band Voltage Determination on the Capacitance-Voltage Characteristic Inflection Point

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***Abstract* – Theoretical analysis of relation between the position of the inflection point on the capacitance-voltage characteristic of an ideal (without interface states) metal-insulator-semiconductor structure and its flat-band voltage is provided. The inflection point position is shown to depend on the relation between semiconductor and insulator capacitances and cannot be a universal characteristic. The inflection point and the flat-band point are found to be equal only when the insulator capacitance coincides with the semiconductor flat-band capacitance. The method of determining the true flat-band voltage position on the normalized capacitance value in the inflection point is proposed.**

***Index Terms* – Flat-band voltage, flat-band capacitance, capacitance-voltage characteristic, inflection point, second derivative.**

I. INTRODUCTION

FLAT-BAND VOLTAGE (V_{FB}) is a very important parameter in physics of metal-insulator-semiconductor structures (MIS) which are widely used in modern electronics. V_{FB} is voltage at which the surface band bending of structure equals zero. The parameter significantly influences the MIS-transistor threshold voltage. There are a lot of methods to determine the parameter. High frequency capacitance-gate voltage characteristics, C(V_G), are often used. In paper [1] a good review of certain methods for determining V_{FB} is presented. The authors of paper [2] suggest the method for determining V_{FB} on the inflection point of C(V_G), C(V_{infl}). They show, with a number of examples, that C(V_{infl}) and the flat-band point, C(V_{FB}), are practically equal and the equality does not depend on the type of a structure, frequency of a test signal and other factors. But in paper [3] the C(V_{infl}) deviation from C(V_{FB}) is shown with the calculated and experimental examples to exist and depend on the dopant level and the insulator capacitance.

The aim of this paper is determining the relation between C(V_{infl}) and C(V_{FB}) of an ideal MIS-structure (without interface states).

II. DETERMINING THE INFLECTION POINT

Introduce the expressions for a series connection of an insulator and semiconductor:

$$\begin{gathered} Vg(y) = Qs(y)/Cd + y; \\ \dot{Q}s(y) = \frac{\partial Qs(y)}{\partial y} \equiv Cs(y), \\ \dot{V}g(y) = \frac{\partial Vg(y)}{\partial y} = \frac{\dot{Q}s(y)}{Cd} + 1 = \frac{Cs(y)}{Cd} + 1; \end{gathered} \tag{1}$$

$$Cg(y) = \frac{dQs}{dVg} = \frac{\dot{Q}s(y)}{\dot{V}g(y)} = \frac{Cs(y)Cd}{Cs(y)+Cd}; \tag{2}$$

where $y = q\varphi_s / kT$ is the normalized band bending (potential) on the semiconductor surface, $Qs(y)$ is the semiconductor charge dependence on the surface potential, Cd is the insulator capacitance, Cs is the semiconductor capacitance, Cg is the structure capacitance, $Vg(y)$ is the normalized potential on the gate of a structure.

To determine the position of C(V_{infl}), it is necessary to calculate the second derivative d^2Cg/dVg^2 and find its zero. To do it, use the formulas for differentiation of functions given in terms of a parameter t [4]:

$$\begin{cases} x = x(t), y = y(t); \\ (\dot{x},\dot{y}) = \dfrac{d}{dt}(x,y), \quad (\ddot{x},\ddot{y}) = \dfrac{d^2}{dt^2}(x,y); \\ y' = \dfrac{dy}{dx} = \dfrac{\dot{y}}{\dot{x}}; \\ y'' = \dfrac{d^2y}{dx^2} = \dfrac{y'}{\dot{x}}\left(\dfrac{\ddot{y}}{\dot{y}} - \dfrac{\ddot{x}}{\dot{x}}\right) = \dfrac{y'}{\dot{x}}\dfrac{d}{dt}\left(\ln(y')\right). \end{cases} \tag{3}$$

In our case: $t \to y$, $x(t) \to Vg(y)$, $y(t) \to Cg(y)$ and using (3) gives:

$$\dot{V}g(y) = \frac{Cs(y)}{Cd} + 1, \; \dot{C}g(y) = \frac{\dot{C}s(y)\cdot Cd^2}{(Cs(y)+Cd)^2},$$

$$Cg'(y) \equiv \frac{dCg}{dVg} = \frac{\dot{C}g(y)}{\dot{V}g(y)} = \dot{C}s(y)\left(\frac{Cd}{Cs(y)+Cd}\right)^3.$$

And from the expression for the second derivative:

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$$Cg''(y) \equiv \frac{d^2Cg}{dVg^2} = \frac{Cg'(y)}{\dot{V}g(y)} \frac{d}{dy}\left(\ln \frac{\dot{C}g(y)}{\dot{V}g(y)} \right) =$$
$$= \dot{C}s(y) \left(\frac{Cd}{Cd + Cs(y)} \right)^4 \left(\frac{\ddot{C}s(y)}{\dot{C}s(y)} - 3 \frac{\dot{C}s(y)}{Cd + Cs(y)} \right),$$

the inflection point condition is obtained:

$$\frac{d^2Cg}{dVg^2} = 0 \Rightarrow Cd = 3\frac{\dot{C}s^2(y)}{\ddot{C}s(y)} - Cs(y). \quad (4)$$

In summary, the C(V_{infl}) position depends on the relation of semiconductor and insulator capacitances and cannot be a universal characteristic.

III. Equality Conditions for Inflection and Flat-Band Points

If the series expansion for function $Q_s(y)$ about y=0 has a form:

$$Qs(y) = Cfb \cdot y \cdot \left(1 + a \cdot y + b \cdot y^2\right),$$

then

$$Cs(y) = Cfb \cdot \left(1 + 2a \cdot y + 3b \cdot y^2\right),$$
$$\dot{C}s(y) = Cfb(2a + 6b \cdot y), \quad \ddot{C}s(y) = Cfb \cdot 6b, \quad (5)$$

there Cfb is the flat-band semiconductor capacitance.

Inserting expressions (5) into (4) gives

$$Cd = Cfb \cdot \left(2a^2/b - 1 + 10a \cdot y + 15b \cdot y^2\right).$$

It was obtained from the last expression at $y = 0$:

$$Cd = Cfb \cdot \left(2a^2/b - 1\right),$$

i.e. C(V_{infl}) and C(V_{FB}) are equal at only a certain insulator capacitance value.

In the classical case of monopolar semiconductor (nondegenerate statistics of charge carriers, absence of interface states, etc.), at depletion/accumulation conditions, the charge dependence on the band bending looks like:

$$Qs(y) = Cfb \cdot sign(y)\sqrt{2}\sqrt{\exp(y) - y - 1} \approx$$
$$\approx Cfb \cdot y \cdot \left(1 + y/6 + (y/6)^2\right), \quad (6)$$

i.e., $a = 1/6, b = a^2 = 1/36 \quad \Rightarrow \quad Cd = Cfb$.

So, in the classical case C(V_{infl}) and C(V_{FB}) are equal only when the insulator capacitance and the flat-band capacitance of a semiconductor are equal: that is when $Cg_{infl} = 0.5 \cdot Cd$. At other conditions, expression $2a^2/b$ can have another value and, consequently, a C(V_{infl}) and C(V_{FB}) equality can happen at another insulator capacitance value.

Considering the examples given in [2] one can notice that $Cg_{infl} \approx 0.5 \cdot Cd$.

IV. The True Flat-Band Voltage Determination

To estimate the C(V_{infl}) deviation from C(V_{FB}) in the classical case the expression (6) is used (without series expansion).

It follows from (1) and (6) that:

$$Cs(y) = Cfb \cdot sign(y) \frac{e^y - 1}{\sqrt{2}\sqrt{e^y - 1 - y}}. \quad (7)$$

Expression (7) was inserted into (4). The insulator capacitance value $Cd_{infl}(y)$ at which the inflection of C(V_G), which is to happen at a certain band bending value, was given with a number of changes:

$$\frac{Cd_{infl}(y)}{Cfb} = sign(y)\sqrt{2} \cdot$$
$$\cdot \frac{e^y\sqrt{e^y - y - 1}\left(e^{2y} - 4e^y y + 4e^y - 2y - 5\right)}{e^{3y} - 2e^{2y}y - 5e^{2y} + 4e^y y^2 + 2e^y y + 7e^y - 3}$$

The property of this expression is that, at $y \to 0$, it has indeterminacy form 0/0 in the fifth power. Therefore, at a small value of y, one can use series expansion:

$$Cd_{infl}(y)/Cfb \approx 1 + 3y/5 + 157y^2/900.$$

The dependence of Cd_{infl}/Cfb on the normalized band bending value y_{infl}, at which the inflection is to happen, is shown in Fig.1. The dependence is approximated by the $\exp(0.6 \cdot y_{infl})$ function which is shown with a small dotted line.

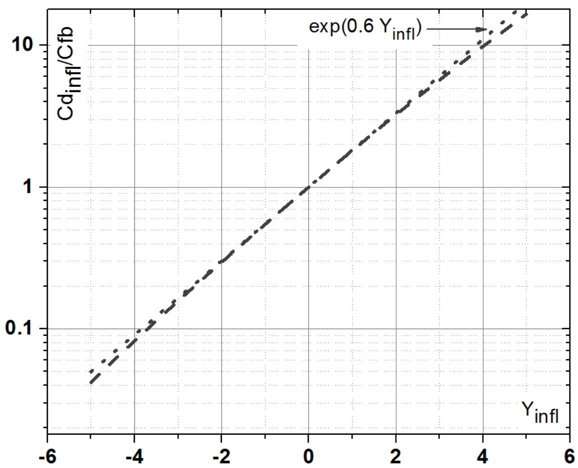

Fig. 1. Ratio Cd_{infl}/Cfb depending on the band-bending in C(V_{infl}). Function $\exp(0.6 y_{infl})$ is shown by a small dotted line.

To determine the normalized structure capacitance values at V_{infl}: Cg_{infl}/Cd, and the normalized capacitance of structure at V_{FB}: $Cg_{fb}(y)/Cd$, expressions (2) and (7) are used:

$$Cg_{infl}(y)/Cd = 1/\left(Cd_{infl}(y)/Cs(y)+1\right),$$
$$Cg_{fb}(y)/Cd = 1/\left(Cd_{infl}(y)/Cfb+1\right).$$

These dependences are shown in Fig.2 and Fig.3.

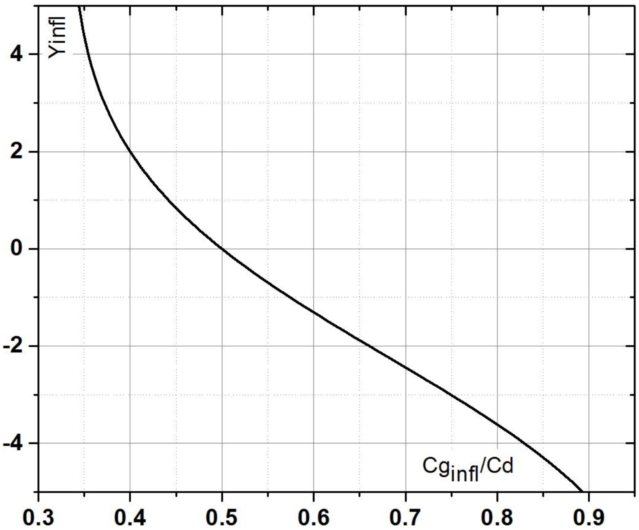

Fig. 2. Dependence of the band bending in the inflection point, $y_{infl} = q\varphi_{infl}/kT$, on the normalized capacitance of the structure at V_{infl}, Cg_{infl}/Cd.

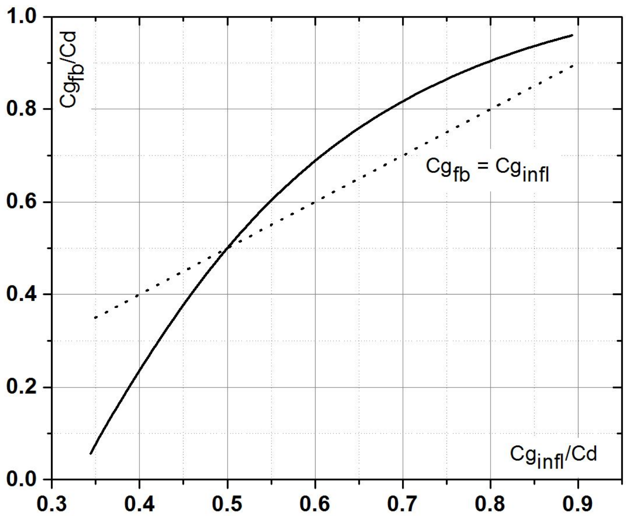

Fig. 3. Dependence of the normalized structure capacitance at V_{FB} Cg_{fb}/Cd on the normalized structure capacitance at V_{infl}, Cg_{infl}/Cd. The result which could have been obtained with the validity of the hypothesis about the $C(V_{infl})$ and $C(V_{FB})$ equality [2] is shown by a dotted line.

Dependence shown on Fig.3 is well fitted by cubic polynomial function at parameter values represented in Table I:

$$Y(x) = a + bx + cx^2 + dx^3$$

where $Y(x) = Cg_{fb}/Cd(x),\ x = Cg_{infl}/Cd$.

TABLE I
CUBIC POLYNOMIAL FITTING COEFFICIENTS

Parameter	a	b	c	d
Parameter value	-1.81721	7.64247	-7.22043	2.40048

Within the classical semiconductor capacitance model considered, these plots and the fitting result allow one to determine the surface band bending, the true flat-band capacitance and, consequently, V_{FB} over the structure capacitance value in the inflection point of capacitance-voltage characteristic.

V. CONCLUSION

The $C(V_{infl})$ and $C(V_{FB})$ equality is possible only at certain insulator capacitance value. This value is fully determined by the semiconductor charge dependence on the surface band bending. In the classical case of an ideal MIS-structure (nondegenerate statistics of charge carriers, absence of interface states, etc.) the equality is possible when the insulator capacitance is equal to the semiconductor capacitance at flat-band, i.e. when structure capacitance value in $C(V_{infl})$ is equal to half of the insulator capacitance.

The method of determining the true flat-band point position on the normalized capacitance value in the inflection point is proposed.

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