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Two-dimensional phononless VRH conduction in arrays of Ge/Si quantum dots

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We report measurements of a two-dimensional variable-range hopping conductance in delta-doped Ge/Si heterostructures with a layer of Ge nanometer-scale quantum dots. We found that the conductance σ vs. temperature T follows the Efros-Shklovskii behavior $\sigma = \sigma_0 \exp[-(T_0/T)^{1/2}]$ with the temperature-independent prefactor $\sigma_0 \sim e^2/h$. A strong reduction of the measured value of T_0 from that calculated for single-particle hopping was observed. All these results provide a manifestation of interaction-driven many-electron correlated hopping in dense arrays of quantum dots.

1 Introduction In general, the temperature dependence of the conductivity for variable-rangle hopping (VRH) is given by $\sigma(T) = \sigma_0(T) \exp[-(T_0/T)^x]$. VRH conductivity in the presence of long-range Hartree interaction between localized single-particle excitations obeys the Efros-Shklovskii (ES) law [1] $\sigma(T) = \sigma_0 \exp[-(T_0/T)^{1/2}]$, where $k_B T_0 = Ce^2/\kappa\xi$ is the characteristic interaction energy scale, C is a numerical coefficient that depends on dimensionality, k_B is the Boltzmann constant, κ is the relative permittivity of the host lattice, ξ is the localization length of electrons. Within the mechanism of phonon-assisted VRH, the prefactor σ_0 takes the form $\sigma_0 = \gamma/T^m$, where γ is a temperature-independent parameter and $m \sim 1$ [2]. The theoretical value of the constant C for single-particle hopping in two dimensions (2D) is $C \simeq 6$ [2, 3].

Several authors have argued that under certain conditions dc VRH conduction can be dominated by many-particle Coulomb correlations between electronic transitions [4, 5]. Sequential correlations appear when the hops of an electron facilitates the hopping probability of another electron due to rearrangement of the local potentials and/or site occupations in the vicinity of the initial and final states for tunneling process. There can be also interaction-driven simultaneous hopping of several electrons resulting in a lowering of the energy configuration of the system. Because formation of such dressed polaron state provides partial screening of Coulomb interaction at large distances, the characteristic interaction energy in correlated hopping is reduced relative to its single-particle value [6, 7]. Pérez-Garrido et al. [4] showed that in a regime of many-particle excitations, $\sigma(T)$ dependence has the ES form with numerical constant $C = 0.6 \pm 0.2$, i.e., the parameter T_0 turns out to be about one order of magnitude smaller than Efros and Shklovskii's prediction for single-electron hoping. A reduction of hopping constants from the single-particle value in gated GaAs/AlGaAs heterostructures has been observed in [8]. Kozub, Baranovskii and Shlimak, assuming that interaction-assisted fluctuations of energies of hopping sites have spectral density

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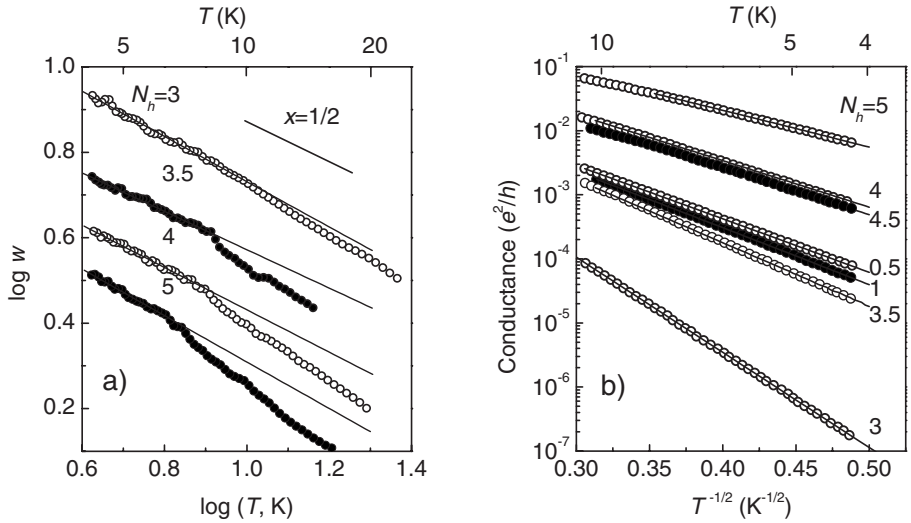

Fig. 1 a) Temperature dependence of the logarithmic derivative $w(T) = \partial \ln \sigma(T)/\partial \ln T$ for samples with different average numbers of holes per one Ge QD. The solid lines are least-square fits to linear dependence. b) The conductance $\sigma(T)$ vs $T^{-1/2}$ for different dot occupation. Symbols correspond to experimental data, solid lines are approximations of the experimental data using equation $\sigma(T) = \gamma/T^m \exp[-(T_0/T)^{1/2}]$. T_0, γ, and m are variable parameters.

$1/f$, demonstrated that sequential Coulomb correlations in a Coulomb glass can result in a phononless VRH with a temperature-independent universal prefactor $\sigma_0 \approx e^2/h$ [5].

Usually, gated disordered semiconductors are exploited to look for the many-particle correlations in 2D VRH. To drive conductivity of the system, one is obliged to change carrier concentration thereby approaching inevitably the metal-insulator transition (MIT). However, since localization degrades the screening of electron-electron interaction, correlation effects should be particularly important on the insulator side far from the MIT. From this point of view, we believe that it is more reasonable to use dense arrays of quantum dots (QDs) to study correlated hopping because one can fix QDs density and change only carrier wavefunctions by varying the dot filling factor, being deep in the insulator phase.

Previously, using an artificial screening provided by a metallic plane, parallel to a layer of Ge QDs in Si, we have proved that the VRH transport in arrays of self-assembled Ge/Si(001) QDs is strongly affected by long-range interactions [9]. In this paper, we examine in detail measurements of 2D variable-range hopping conductance in delta-doped Ge/Si heterostructures with a layer of Ge nanometer-scale QDs grown by molecular-beam epitaxy in the Stranskii-Krastanov growth mode. The average size of the dot base length is around 10 nm, the height is ~ 1 nm. The areal density of the dots is $n_{\rm QD} = 4 \times 10^{11}$ cm^{-2}. To supply holes on the dots, a boron δ-doping Si layer inserted 5 nm below the Ge QD layer was grown. After spatial transfer, the average number of holes per dot was varied from $N_h = 1/2$ to $N_h = 5$ by varying the doping. The sample preparation and data analysis are described in detail elsewhere [10].

2 Temperature dependence of conductivity In order to obtain detailed information on the functional dependence $\sigma(T)$, we used the differential method for an analysis of the temperature dependence of the reduced activation energy [11] $w(T) = \partial \ln \sigma(T)/\partial \ln T = m + x(T_0/T)^x$. In this approach, if $m \ll x(T_0/T)^x$, then $\log w(T) = A - x \log T$, and $A = x \log T_0 + \log x$. Plotting $\log w$ as a function of $\log T$, one can find the hopping exponent x from the slope of the straight line. The parameter A can be found by the intersection point of the straight line with the ordinate axis, which gives the characteristic temperature $T_0 = (10^A/x)^{1/x}$. Typical plots of $\log w(T)$ versus $\log T$ for several samples are given in Fig. 1 a. At

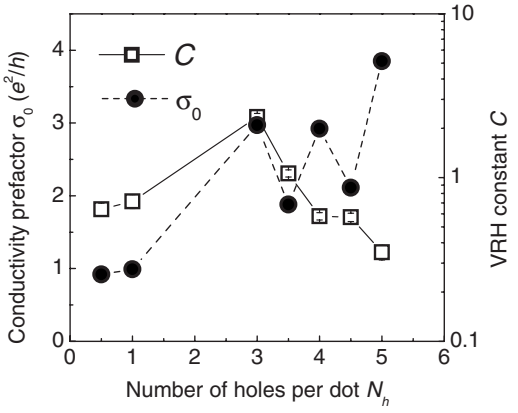

Fig. 2 Dependence of the conductivity prefactor and numerical parameter C associated with variable-range hopping in arrays of Ge/Si quantum dots on the average number of holes in QDs.

$T < 10$ K, a linear relationship is observed between $\log w(T)$ and $\log T$, implying that $m \ll x(T_0/T)^x$ at these temperatures. From the slope angle of the approximating straight lines (solid lines in Fig. 1 a), we found that the exponent x takes approximately the same value $x = 0.51 \pm 0.05$ for all samples.

Because it was already established that $x \simeq 0.5$, the method of non-linear regression can be used for further determining the exponent m in the region of low temperatures. With this aim, the experimental data $\sigma(T)$ at $T < 10$ K were approximated by the equation $\sigma(T) = \gamma/T^m \exp[-(T_0/T)^{1/2}]$, and the parameters γ, m and T_0 were varied to obtain the best fit. Figure 1 b shows the conductivity in units of e^2/h, the quantum of conductance, of samples with different QD occupation plotted versus $T^{-1/2}$; the symbols are the experimental points and the solid lines are the least-squares fits to the ES equation. We found that m lies in the region 0.16 ± 0.09. This means that the conductivity prefactor σ_0 virtually does not depend on temperature at low T and signals *against* the conventional phonon-assisted hopping mechanism.

The dependence $\sigma_0(N_h)$ is presented in Fig. 2. An impressive feature is that the prefactor having a value of order e^2/h is not constant but quantized in units of the conductance quantum. Although, currently, there is no preconceived explanation of the oscillating behavior of σ_0, we consider universality of the prefactor as a manifestation of the 2D VRH conduction stimulated by the sequential Coulomb correlations [5].

To obtain further evidence for correlated VRH in arrays of Ge/Si QDs, it is necessary to measure the ES characteristic temperature T_0 and compare it with the theoretical predictions. In fact, since T_0 depends on the localization length ξ and hence on the electronic configuration of occupied hole state in the dots, it is more convenient to find the universal constant C whose value is inverse proportional to intensity of the many-particle effects. It may be done taking into account the values of T_0 and ξ.

3 Analysis of the parameters associated with VRH in Ge/Si QDs Asymptotic values of the hole localization length resulting from hole tunneling between coupled Ge/Si quantum dots were obtained by computer modeling. The simulation was performed on a square lattice of 15×15 sites with the lattice constant $n_{\mathrm{QD}}^{-1/2} + \delta r$, where δr is a random value with a Gaussian distribution. Only overlapping between nearest neighbours were included. We use the Hamiltonian

$$\hat{H} = \sum_{i,\alpha} E_{i,\alpha} \hat{a}^{+}_{i,\alpha} \hat{a}_{i,\alpha} + \sum_{i,j,\alpha,\beta} J_{i,j,\alpha,\beta} \hat{a}^{+}_{j,\beta} \hat{a}_{i,\alpha}, \tag{1}$$

where index i counts the dots, index α denotes the hole bound state number in QD (we consider only nine bound states in each dot); $\hat{a}^{+}_{i,\alpha}$ ($\hat{a}_{i,\alpha}$) the creation (annihilation) operator for a hole in state α of ith QD, $E_{i,\alpha}$ is the hole energy in this state, $J_{i,j,\alpha,\beta}$ is the integral of overlapping between the αth state in ith

QD and βth state in jth QD. The random hole energies $E_{i,\alpha}$ were taken as the size-quantization energies in quantum dots whose dimensions are characterized by a Gaussian distribution with the mean square deviation of 20%. Dependence of the energy levels on dot size has been calculated previously using the sp^3 tight binding model with inclusion of spin-orbit interaction and deformation effects [12].

The following procedure was used to determine the overlap integrals. We calculated energies of hole states in a model structure containing Ge quantum dot inside a Si box. Periodic boundary conditions, $\psi(-d/2, y, z) = \psi(d/2, y, z)$ or $\psi(-d/2, y, z) = -\psi(d/2, y, z)$, where d is the box size in x direction, $\psi(x, y, z)$ the hole wave function, were considered. The same boundary conditions were used for the y and z directions. Overlap integral in plane of QD array in x direction was defined as $J(d) = |E_+ - E_-|/4$, where E_+ and E_- are the hole energies corresponding to the boundary conditions given above. The obtained dependence $J(d)$ can be rewritten in the form $J(d) = A_\alpha \exp(-B_\alpha d)$, where coefficients A_α and B_α depend on the energy level number α, and B_α equals to the inverse localization length of a hole in α state of isolated QD. Integrals $J_{i,j,\alpha,\beta}$ were determined as geometrical mean of overlap integrals between α and β states: $J_{i,j,\alpha,\beta} = \sqrt{J_{i,j,\alpha,\alpha} J_{i,j,\beta,\beta}} = \sqrt{A_\alpha A_\beta} \exp[-(B_\alpha + B_\beta) d_{ij}/2]$, where $d_{i,j}$ is the distance between QDs. Simulation was carried out using 5000 random realizations of QD array with the filling factor 1/2 for the ground s-state or for the first excited p-state in the dots. For each realization, we calculated probability p_i of hole to occupy corresponding s- or p-state in each dot. The probability value was then approximated by the equation $p_i = a \exp(-2d_i/\xi_r)$, where d_i is the distance between ith QD and the dot with maximum local hole wavefunction amplitude. Localization length ξ was obtained by averaging ξ_r through all array realizations. We found $\xi = 2.3$ nm for $N_h \leq 2$ and $\xi = 2.8$ nm for $2 < N_h \leq 6$.

It now remains to determine the magnitude of C using equation $C = k_B T_0/(e^2/\kappa\xi)$ and experimental values of T_0. These results are presented in Fig. 2. The value of hopping constant turns out to be $C = 0.9 \pm 0.4$. This implies that characteristic interaction temperature T_0 is considerably smaller than the value from simulation of single-electron transport that provides an additional argument in favor of correlated hopping in 2D arrays of Ge/Si quantum dots.

4 Summary In summary, we have investigated the variable-range hopping transport of holes in Ge/Si self-assembled quantum dots. We find the universal temperature-independent conductivity prefactor $\sigma_0 \sim e^2/h$ and demonstrate the hopping constant C to be much smaller that the single-particle value. We believe that our results provide experimental evidence for many-electron correlated hopping in two-dimensional arrays of quantum dots.

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